

Nonclassicality and decoherence of photon-added squeezed thermal state in thermal environment

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Theoretical analysis is given of nonclassicality and decoherence of the field states generated by adding any number of photons to the squeezed thermal state (STS). Based on the fact that the squeezed number state can be considered as a single-variable Hermite polynomial excited state, the compact expression of the normalization factor is derived, a Legendre polynomial. The nonclassicality is investigated by exploring the sub-Poissonian and negative Wigner function (WF). The results show that the WF of single photon-added STS (PASTS) always has negative values at the phase space center. The decoherence effect on PASTS is examined by the analytical expression of WF. It is found that a longer threshold value of decay time is included in single PASTS than in single-photon subtraction STS.

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I. INTRODUCTION

Generation and manipulation of non-classical light field has been a topic of great interest in quantum optics and quantum information science [1]. Many experimental schemes have been proposed to generate nonclassical states of optical field. Among them, subtracting photons from and/or adding photons to quantum states have been paid much attention because these fields exhibit an abundant of nonclassical properties and may give access to a complete engineering of quantum states and to fundamental quantum phenomena [2–10]. For example, quantum-to-classical transition has been realized experimentally through single-photon-added coherent states of light. These states allow one to witness the gradual change from the spontaneous to the stimulated regimes of light emission [4]. For m -photon-added coherent state in the dissipative channel, the nonclassical properties are studied theoretically [11] by deriving the analytical expression of the Wigner function (WF), which turns out to be a Laguerre-Gaussian function. As another example, photon addition and subtraction experimentally have been employed to probe quantum commutation rules by Parigi *et al.* In fact, they have implemented simple alternated sequences of photon creation (addition) and annihilation (subtraction) on a thermal field and observed the noncommutativity of the creation and annihilation operators [6]. In addition, photon subtraction/addition can be applied to improve entanglement between Gaussian states [12, 13], loophole-free tests of Bell's inequality [14, 15], and quantum computing [16].

On the other hand, it is interesting to notice that subtracting or adding one photon from/to pure squeezed vacuum can generate the same output state, i.e., squeezed single-photon state [17]. Actually, the photon addition is able to generate a nonclassical state (e.g coherent and thermal states), which is quite different from photon subtraction only from a nonclassical state [18–20]. In addition,

the resulting states obtained by successive photon subtractions or additions are different from each other. For instance, successive two-photon additions $[a^{\dagger 2}]$ and successive two-photon subtractions $[a^2]$ will result in the same state produced by using subtraction-addition ($a^{\dagger}a$) and addition-subtraction (aa^{\dagger}), respectively. In Ref.[21], two photon-subtracted squeezed vacuum is used to generate the squeezed superposition of coherent states with high fidelities and large amplitudes.

In general, different non-Gaussian operators (e.g subtracting and adding photon) will suffer different effects from the surroundings, thus it is important to know which operator is more robust compared to the other under an identical initial quantum state when the environment is taken into account. Very recently, the robustness of several superposition states is studied by using the linear entropy under a thermal environment [22]. In this paper, we shall introduce a kind of nonclassical state—photon-addition squeezed thermal state (PASTS), generated by adding photon to squeezed thermal state (STS) which can be considered as a generalized Gaussian state. Then we shall investigate the nonclassical properties and decoherence of single-mode any number PASTS under the influence of thermal environment.

This paper is organized as follows. In Sec. II we introduce the single-mode PASTS. By converting the PASTS to an Hermite polynomial excitation squeezed vacuum state, we derive a compact expression for the normalization factor of PASTS, which is an m -order Legendre polynomial of squeezing parameter λ and mean number n_c of thermal state, where m is the number of added photons. In Sec III, we discuss the nonclassical properties of the PASTS in terms of sub-Poissonian statistics and the negativity of its WF. We find the negative region of WF in phase space and there is an upper bound value of λ for this state to exhibit sub-Poissonian statistics which increases as m increases. Then, in Sec. IV we derive the explicitly analytical expression of time evolution of WF of the arbitrary PASTS in the thermal channel and discuss

the loss of nonclassicality in reference of the negativity of WF. The threshold value of decay time corresponding to the transition of the WF from partial negative to completely positive definite is obtained at the center of the phase space, which is independent of parameters λ and n_c . It shown that the WF for single PASTS (SPASTS) has always negative value for all parameters λ and n_c if the decay time $\kappa t < \frac{1}{2} \ln[(2\mathcal{N} + 2)/(2\mathcal{N} + 1)]$ (see Eq.(46) below), where \mathcal{N} denotes the average thermal photon number in the environment with dissipative coefficient κ . Comparing to the case of single photon-subtraction STS (SPSSTS), the decoherence time of SPASTS is longer. In this sense, the photon-addition non-Gaussian states present more robust contrast to decoherence than photon-subtraction ones. The reason may be that the amount of non-Gaussianity for SPASTS is larger than that for SPSSTS as presented in Sec. V. Conclusions are involved in the last section.

II. PHOTON-ADDITION SQUEEZED THERMAL STATE (PASTS)

The m -photon-added scheme, denoted by the mapping $\rho \rightarrow a^{\dagger m} \rho a^m$, was first proposed by Agarwal and Tara [18]. Here, we introduce the PASTS. Theoretically, the PASTS can be obtained by repeatedly operating the photon creation operator a^\dagger on a STS, so its density operator is given by

$$\rho_{ad} = C_{a,m}^{-1} a^{\dagger m} S_1^\dagger \rho_{th} S_1 a^m, \quad (1)$$

where m is the added photon number (a non-negative integer), $C_{a,m}^{-1}$ is the normalization constant to be determined, and $S_1 = \exp[\lambda(a^2 - a^{\dagger 2})/2]$ is the single-mode squeezing operator with λ being squeezing parameter [23, 24]. ρ_{th} is a single field mode with frequency ω in a thermal equilibrium state corresponding to absolute temperature T , whose the density operator is [25]

$$\rho_{th} = \sum_{n=0}^{\infty} \frac{n_c^n}{(n_c + 1)^{n+1}} |n\rangle \langle n| = \frac{1}{n_c} :e^{-\frac{1}{n_c} a^\dagger a}:, \quad (2)$$

($: \cdot :$ denoting antinormally ordering) which implies that the density operator ρ_{th} can be expanded as

$$\rho_{th} = \frac{1}{n_c} \int \frac{d^2 \alpha}{\pi} e^{-\frac{1}{n_c} |\alpha|^2} |\alpha\rangle \langle \alpha|, \quad (3)$$

where $n_c = [\exp(\omega/(kT)) - 1]^{-1}$ being the average photon number of the thermal state ρ_{th} and k_B being Boltzmann's constant. Eq.(3) is useful for later calculation.

A. Squeezed number state as a Hermite polynomial excited state

Recalling that the single-mode squeezed operator S_1 has its natural expression in the coordinate representa-

tion [26],

$$S_1 = \frac{1}{\sqrt{\mu}} \int_{-\infty}^{\infty} dq \left| \frac{q}{\mu} \right\rangle \langle q|, \mu = e^\lambda, \quad (4)$$

where $|q\rangle$ is the eigenstate of $Q = (a + a^\dagger)/\sqrt{2}$, $Q|q\rangle = q|q\rangle$, and

$$|q\rangle = \pi^{-1/4} \exp \left\{ -\frac{q^2}{2} + \sqrt{2} q a^\dagger - \frac{a^{\dagger 2}}{2} \right\} |0\rangle. \quad (5)$$

Thus, using Eq.(5) and the overlap relation

$$\langle q|n\rangle = \frac{1}{\sqrt{2^n n! \sqrt{\pi}}} e^{-q^2/2} H_n(q), \quad (6)$$

where $H_n(q)$ is the single-variable Hermite polynomial then $S_1|n\rangle$ can be expressed as

$$\begin{aligned} S_1|n\rangle &= \int_{-\infty}^{\infty} \frac{dq}{\sqrt{2^n n! \sqrt{\pi}}} e^{-q^2/2} H_n(q) \left| \frac{q}{\mu} \right\rangle \\ &= \frac{\text{sech}^{1/2} \lambda}{\sqrt{2^n n!}} \frac{\partial^n}{\partial \tau^n} e^{\sqrt{2} a^\dagger \tau \text{sech} \lambda + (\tau^2 - \frac{1}{2} a^{\dagger 2}) \tanh \lambda} |0\rangle \Big|_{\tau=0} \\ &= \frac{(i\sqrt{\tanh \lambda})^n}{\sqrt{2^n n!}} H_n \left(\frac{a^\dagger \text{sech} \lambda}{i\sqrt{2} \tanh \lambda} \right) S_1|0\rangle, \end{aligned} \quad (7)$$

where we have set $\text{sech} \lambda = 2\mu/(\mu^2 + 1)$, $\tanh \lambda = (\mu^2 - 1)/(\mu^2 + 1)$, and we have used $S_1|0\rangle = \text{sech}^{1/2} \lambda \exp[-a^{\dagger 2}/2 \tanh \lambda] |0\rangle$ as well as the generating function of $H_n(q)$ [27]:

$$H_n(q) = \frac{\partial^n}{\partial \tau^n} \exp(2q\tau - \tau^2) \Big|_{\tau=0}. \quad (8)$$

Eq.(7) indicates that the single-mode squeezed number state $S_1|n\rangle$ is actually a Hermite polynomial excited squeezed vacuum state [28]. Obviously, when $n = 0$, $H_0(q) = 1$, Eq.(7) just reduces to single-mode squeezed vacuum. While for $n = 1, 2$, noting $H_1(q) = 2q$ and $H_2(q) = 4q^2 - 2$, Eq.(7) become

$$\begin{aligned} S_1|1\rangle &= a^\dagger \text{sech} \lambda S_1|0\rangle, \\ S_1|2\rangle &= \frac{1}{\sqrt{2}} (a^{\dagger 2} \text{sech}^2 \lambda + \tanh \lambda) S_1|0\rangle, \end{aligned} \quad (9)$$

respectively. It is interesting to notice that the single photon-added squeezed vacuum (PASV) is equal to the squeezed number state $S_1|1\rangle$, and the two PASV can be considered as a superposition of the squeezed number state $S_1|2\rangle$ and the squeezed vacuum.

B. Normalization of PASTS

To fully describe a quantum state, its normalization is usually necessary. Next, we shall employ the fact (7) to realize our aim. First, let us derive the normally ordering

form of STS $\rho_s \equiv S_1^\dagger \rho_{th} S_1$, which is convenient for further calculation of normalization.

Using Eqs.(2) and (7), we can rewrite the STS ρ_s as

$$\begin{aligned} \rho_s &= \sum_{n=0}^{\infty} \frac{n_c^n}{(n_c+1)^{n+1}} S_1(-\lambda) |n\rangle \langle n| S_1^\dagger(-\lambda) \\ &= \frac{\text{sech}\lambda}{n_c+1} \sum_{n=0}^{\infty} \frac{(n_c \tanh \lambda)^n}{2^n n! (n_c+1)^n} : H_n \left(\frac{-a^\dagger \text{sech}\lambda}{\sqrt{2 \tanh \lambda}} \right) \\ &\quad \times \exp \left[\frac{1}{2} (a^2 + a^{\dagger 2}) \tanh \lambda - a^\dagger a \right] H_n \left(\frac{-a \text{sech}\lambda}{\sqrt{2 \tanh \lambda}} \right) : , \end{aligned} \quad (10)$$

where $S_1^\dagger(-\lambda) = S_1(\lambda)$ and the vacuum projector $|0\rangle \langle 0| =: \exp[-a^\dagger a] :$ is used. Further using the two-linear generating function of Hermite polynomial [29],

$$\begin{aligned} &\sum_{n=0}^{\infty} \frac{t^n}{2^n n!} H_n(x) H_n(y) \\ &= \frac{1}{\sqrt{1-t^2}} \exp \left[\frac{2txy - t^2(x^2 + y^2)}{1-t^2} \right] , \end{aligned} \quad (11)$$

we can directly obtain the normally ordering form of STS,

$$\rho_s = \frac{1}{\sqrt{A}} : \exp \left[\frac{C}{2} (a^{\dagger 2} + a^2) + (B-1) a^\dagger a \right] : , \quad (12)$$

where we have set

$$\begin{aligned} A &= n_c^2 + (2n_c+1) \cosh^2 \lambda, \\ B &= \frac{n_c}{A} (n_c+1), \\ C &= \frac{2n_c+1}{2A} \sinh 2\lambda. \end{aligned} \quad (13)$$

By introducing $a = (Q + iP)/\sqrt{2}$ and $a^\dagger = (Q - iP)/\sqrt{2}$, Eq.(12) can be put into another form

$$\rho_s = \frac{1}{\tau_1 \tau_2} : \exp \left[-\frac{Q^2}{2\tau_1^2} - \frac{P^2}{2\tau_2^2} \right] : , \quad (14)$$

where $\tau_1 \tau_2 = \sqrt{A}$, and

$$\begin{aligned} 2\tau_1^2 &= (2n_c+1) e^{2\lambda} + 1, \\ 2\tau_2^2 &= (2n_c+1) e^{-2\lambda} + 1. \end{aligned} \quad (15)$$

Eq.(12) or (14) is a compact expression of the STS, which is just a Gaussian distribution within normal order for operators Q and P [30].

Next, we shall derive the normalization factor for PASTS. Employing Eq.(12), the PASTS reads as

$$\rho_{ad} = \frac{C_{a,m}^{-1}}{\tau_1 \tau_2} : a^{\dagger m} \exp \left[\frac{C}{2} (a^{\dagger 2} + a^2) + (B-1) a^\dagger a \right] a^m : . \quad (16)$$

Thus the normalization factor $C_{a,m}$ is ($1 = \text{tr} \rho_{ad}$)

$$\begin{aligned} C_{a,m} &= \frac{1}{\tau_1 \tau_2} \int \frac{d^2 \alpha}{\pi} |\alpha|^{2m} e^{-(1-B)|\alpha|^2 + \frac{C}{2}(\alpha^{*2} + \alpha^2)} \\ &= \frac{\partial^{2m}}{\partial s^m \partial t^m} \int \frac{d^2 \alpha}{\pi \tau_1 \tau_2} e^{-(1-B)|\alpha|^2 + s\alpha^* + t\alpha + \frac{C}{2}(\alpha^{*2} + \alpha^2)} \Big|_{s=t=0} \\ &= \frac{\partial^{2m}}{\partial s^m \partial t^m} \exp \left[A(1-B)st + \frac{AC}{2}(s^2 + t^2) \right] \Big|_{s=t=0}, \end{aligned} \quad (17)$$

where we have used the completeness relation of coherent state, and $[(1-B)^2 - C^2]^{-1} = \tau_1^2 \tau_2^2 = A$, as well as the integration formula [31]

$$\begin{aligned} &\int \frac{d^2 z}{\pi} \exp \left(\zeta |z|^2 + \xi z + \eta z^* + f z^2 + g z^{*2} \right) \\ &= \frac{1}{\sqrt{\zeta^2 - 4fg}} \exp \left[\frac{-\zeta \xi \eta + \xi^2 g + \eta^2 f}{\zeta^2 - 4fg} \right] , \end{aligned} \quad (18)$$

whose convergent condition is $\text{Re}(\zeta \pm f \pm g) < 0$, $\text{Re}(\zeta^2 - 4fg)/(\zeta \pm f \pm g) < 0$.

Recalling the newly found formula of Legendre polynomial [32, 33], i.e.,

$$\begin{aligned} &\frac{\partial^{2m}}{\partial t^m \partial \tau^m} \exp \left(-t^2 - \tau^2 + \frac{2x\tau t}{\sqrt{x^2 - 1}} \right) \Big|_{t,\tau=0} \\ &= \frac{2^m m!}{(x^2 - 1)^{m/2}} P_m(x) , \end{aligned} \quad (19)$$

and noticing $x^2 - 1 = AC^2$, together with $x = \sqrt{A}(1-B) = [A - n_c(n_c+1)]/\sqrt{A}$, we have

$$\begin{aligned} C_{a,m} &= \frac{(AC)^m}{2^m} \frac{\partial^{2m}}{\partial s^m \partial t^m} \exp \left[\frac{2}{C} (1-B)st - s^2 - t^2 \right] \Big|_{s=t=0} \\ &= m! A^{m/2} P_m(\bar{B}/\sqrt{A}) , \end{aligned} \quad (20)$$

which indicates that $C_{a,m}$ is also just related to Legendre polynomial, and

$$\bar{B} = n_c \cosh 2\lambda + \cosh^2 \lambda. \quad (21)$$

It is noted that, for the case of no-photon-addition with $m = 0$, $C_{a,0} = 1$ as expected. Under the case of m -photon-addition thermal state (no squeezing) with $\bar{B} = n_c + 1$, $A = (n_c + 1)^2$, and $P_m(1) = 1$, then $C_{a,m} = m! (n_c + 1)^m$. The same result as Eq.(32) can be found in Ref.[34].

III. NONCLASSICAL PROPERTIES OF PASTS

In this section, we shall discuss the nonclassical properties of PASTS in terms of sub-Poissonian statistics and the negativity of its WF.

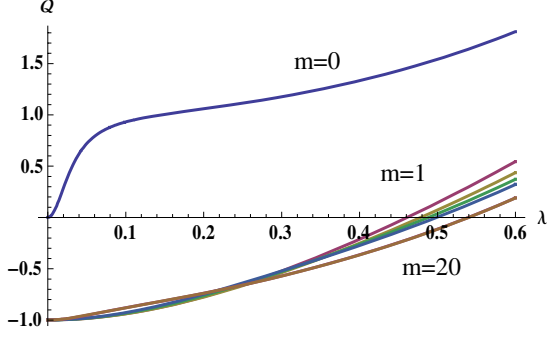


FIG. 1: (Color online) The \mathcal{Q} -parameter as the function of squeezing parameter for different $m = 0, 1, 2, 3, 4, 19, 20$ with a small n_c value.

A. Sub-Poissonian nature of PASTS

The nonclassicality of the PASTS can be analyzed by studying its sub-Poissonian distribution. Using Eq.(20) we can directly calculate:

$$\langle a^\dagger a \rangle = \frac{C_{a,m+1}}{C_{a,m}} - 1, \quad (22)$$

$$\langle a^{\dagger 2} a^2 \rangle = \frac{C_{a,m+2}}{C_{a,m}} - 4 \frac{C_{a,m+1}}{C_{a,m}} + 2. \quad (23)$$

Thus the Mandel's \mathcal{Q} -parameter [35] can be obtained by substituting Eqs.(22) into $\mathcal{Q} \equiv \langle a^{\dagger 2} a^2 \rangle / \langle a^\dagger a \rangle - \langle a^\dagger a \rangle$,

$$\mathcal{Q} = \frac{C_{a,m+2} - 4C_{a,m+1} + 2C_{a,m}}{C_{a,m+1} - C_{a,m}} - \frac{C_{a,m+1} - C_{a,m}}{C_{a,m}}. \quad (24)$$

The negativity of the Mandel's \mathcal{Q} -parameter refers to sub-Poissonian statistics of the state. In order to see clearly the variation of \mathcal{Q} -parameter with λ and n_c , we show the plots of \mathcal{Q} -parameter in Fig.1, from which one can clearly see that, for a given small n_c value, \mathcal{Q} -parameter becomes negative ($m \neq 0$) when λ is less than a certain threshold value which increases as m increases; while for $m = 0$ or a large n_c , \mathcal{Q} is always positive. This implies that the nonclassicality is enhanced by adding photon to squeezed state. Here, we should emphasize that the WF has negative region for all λ and n_c , and thus the PASTS is nonclassical.

B. Photon-number distribution (PND) of the PASTS

The photon-number distribution (PND) is a key characteristic of every optical field. For this purpose, we first calculate the PND of STS, then the PND of PASTS can be directly obtain. The PND, i.e., the probability of finding n photons in a quantum state described by the density operator ρ , is $\mathcal{P}(n) = \langle n | \rho | n \rangle$. So the PND of

the STS is

$$\mathcal{P}(n) = \langle n | S_1^\dagger \rho_{th} S_1 | n \rangle. \quad (25)$$

Using the fact in (7) and the P-representation of ρ_{th} (3), Eq.(25) can be directly written as

$$\begin{aligned} \mathcal{P}(n) &= \frac{\text{sech} \lambda}{2^n n! n_c} \frac{\partial^{2n}}{\partial t^n \partial \tau^n} \exp \left[(t^2 + \tau^2) \tanh \lambda \right] \\ &\times \int \frac{d^2 \alpha}{\pi} \exp \left[\sqrt{2} (\alpha t + \alpha^* \tau) \text{sech} \lambda - \frac{n_c + 1}{n_c} |\alpha|^2 \right] \\ &\times \exp \left[-\frac{\tanh \lambda}{2} (\alpha^2 + \alpha^{*2}) \right]_{\tau=t=0} \\ &= \frac{\text{sech} \lambda}{2^n n! \sqrt{A}} \frac{\partial^{2n}}{\partial t^n \partial \tau^n} \exp \left[2Bt\tau + C(t^2 + \tau^2) \right]_{\tau=t=0}. \end{aligned} \quad (26)$$

In a similar way to deriving Eq.(20), using Eq.(19) we have

$$\mathcal{P}(n) = \frac{D^{n/2}}{\sqrt{A}} P_n \left(B/\sqrt{D} \right), \quad (27)$$

where

$$D = \frac{n_c^2 - (2n_c + 1) \sinh^2 \lambda}{n_c^2 + (2n_c + 1) \cosh^2 \lambda}. \quad (28)$$

Eq.(27) shows that the PND of STS is the Legendre polynomial of B/\sqrt{D} . In particular, when $\lambda = 0$, $A = (n_c + 1)^2$ and $B/\sqrt{D} = 1$, $D = n_c^2/(n_c + 1)^2$, then Eq.(27) becomes $\mathcal{P}(n) = n_c^n/(n_c + 1)^n$, corresponding to the PND of thermal state [34]. In fact, we can also check Eq.(27) using the normalization condition. Note that the Legendre polynomial can also be defined as the coefficients in a Taylor series expansion [36]

$$\frac{1}{\sqrt{1 - 2xt + t^2}} = \sum_{n=0}^{\infty} P_n(x) t^n, \quad (29)$$

thus $\sum_{n=0}^{\infty} \mathcal{P}(n) = 1/\sqrt{A(1 - 2B + D)} = 1$ as expected.

Next, we turn to present the PND of PASTS. From Eq.(27) and noting $a^{\dagger m} |n\rangle = \sqrt{(m+n)!/n!} |m+n\rangle$ and $a^m |n\rangle = \sqrt{n!/(n-m)!} |n-m\rangle$, it then directly follows

$$\begin{aligned} \mathcal{P}_2(n) &= C_{a,m}^{-1} \langle n | a^{\dagger m} \rho_s a^m | n \rangle \\ &= \frac{n! C_{a,m}^{-1} D^{(n-m)/2}}{(n-m)! \sqrt{A}} P_{n-m} \left(B/\sqrt{D} \right). \end{aligned} \quad (30)$$

Eq.(30) is the PND of PASTS, a Legendre polynomial with a condition $n \geq m$ which implies that the photon-number (n) involved in PASTS is always no-less than the photon-number (m) operated on the STS, and there is no photon distribution when $n < m$. For some other non-Gaussian states, such as $a^{\dagger n} a^m \rho_s a^{\dagger m} a^n$, $a^m a^{\dagger n} \rho_s a^n a^{\dagger m}$, and $a^m \rho_s a^{\dagger m}$, their PNDs can also be directly obtained

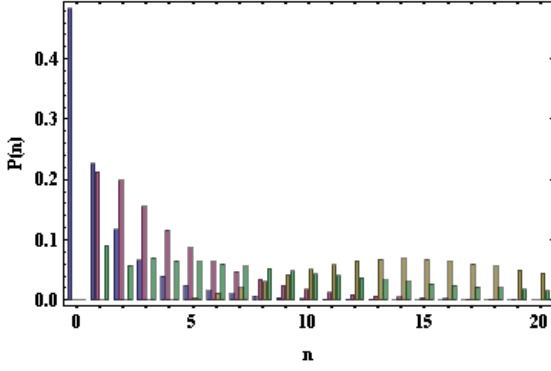


FIG. 2: (Color online) Photon-number distributions of PASTS with $\bar{n}=1$ for $\lambda=0.3$, $m=0$ (blue bar); $\lambda=0.3$, $m=1$ (red bar), $\lambda=0.3$, $m=5$ (yellow bar), and $\lambda=0.8$, $m=1$ (green bar).

by using Eq.(27). In Fig. 2, the PND is shown for different values (λ, n_c) and m . By adding photons, we have been able to move the peak from zero photons to nonzero photons (see blue and red bar in Fig.2). The position of peak depends on how many photons are created and how much the state is squeezed initially. The probability of PND becomes smaller with the increasement of squeezing parameter (see red and green bar in Fig.2).

IV. WIGNER FUNCTION OF PASTS

Next, the normally ordering form Eq.(12) is applied to deduce the WF of PASTS. The partial negativity of WF is indeed a good indication of the highly nonclassical character of the state. Therefore it is worth of obtaining the WF for any states. The WF $W(\alpha, \alpha^*)$ associated with a quantum state ρ can be derived as follows [23]:

$$W(\alpha, \alpha^*) = e^{2|\alpha|^2} \int \frac{d^2\beta}{\pi^2} \langle -\beta | \rho | \beta \rangle e^{2(\alpha\beta^* - \alpha^*\beta)}, \quad (31)$$

where $|\beta\rangle = \exp(-|\beta|^2/2 + \beta a^\dagger) |0\rangle$ is the coherent state.

Substituting Eq.(16) into Eq.(31), we can finally obtain the WF of PASTS (see Appendix A),

$$W(\alpha, \alpha^*) = F_m(\alpha, \alpha^*) W_0(\alpha, \alpha^*), \quad (32)$$

where $W_0(\alpha, \alpha^*)$ is the WF of STS,

$$W_0(\alpha, \alpha^*) = \frac{1}{\pi(2n_c + 1)} \exp \left[-\frac{2 \cosh 2r}{2n_c + 1} |\alpha|^2 + \frac{\sinh 2r}{2n_c + 1} (\alpha^2 + \alpha^{*2}) \right], \quad (33)$$

and

$$F_m(\alpha, \alpha^*) = \frac{(m!)^2 C_{am}^{-1} \sinh^m 2\lambda}{2^{2m} (2n_c + 1)^m} \times \sum_{l=0}^m \frac{(-1)^l 2^{2l} (n_c + \cosh^2 \lambda)^l}{l! [(m-l)!]^2 \sinh^l 2\lambda} |H_{m-l}(\bar{\gamma})|^2, \quad (34)$$

where $\bar{\gamma} = [\alpha^* \sinh 2\lambda - 2\alpha(\cosh^2 \lambda + n_c)] / \{i[(2n_c + 1) \sinh 2\lambda]^{1/2}\}$. Eq.(32) is the analytical expression of WF for PASTS, related to single-variable Hermite polynomials. In particular, when $m = 0$, $F_0(\alpha, \alpha^*) = 1$, Eq.(32) becomes $W(\alpha, \alpha^*) = W_0(\alpha, \alpha^*)$; while for $\lambda = 0$, note $C_{am} = m! (n_c + 1)^m$, $W_0(\alpha, \alpha^*) = e^{-2|\alpha|^2/(2n_c+1)} / [\pi(2n_c+1)]$ and $F_m(\alpha, \alpha^*) = (-1)^m / (2n_c + 1)^m L_m[4(n_c + 1)|\alpha|^2 / (2n_c + 1)]$, Eq.(32) reduces to

$$W(\alpha, \alpha^*) = \frac{(-1)^m e^{-\frac{2|\alpha|^2}{2n_c+1}}}{\pi(2n_c + 1)^{m+1}} L_m \left(\frac{4(n_c + 1)}{2n_c + 1} |\alpha|^2 \right), \quad (35)$$

which corresponds to the WF of m -photon added thermal state [34], and can be checked directly from Eq.(A3). In addition, for $m = 1$, [single-photon-added squeezed thermal state (SPASTS)], $C_{a1} = \bar{B}$ (20), the special WF of SPASTS is

$$W_1(\alpha, \alpha^*) = F_1(\alpha, \alpha^*) W_0(\alpha, \alpha^*), \quad (36)$$

where

$$F_1(\alpha, \alpha^*) = \frac{\sinh 2\lambda}{(2n_c + 1) \bar{B}} \left[|\bar{\gamma}|^2 - \frac{n_c + \cosh^2 \lambda}{\sinh 2\lambda} \right]. \quad (37)$$

Noting $\bar{B} > 0$, thus from Eq.(37) one can see that when the factor $F_1(\alpha, \alpha^*) < 0$, the WF of SPASTS has its negative distribution in phase space. This indicates that the WF of SPASTS always has the negative values at the phase space center $\alpha = 0$ ($\bar{\gamma} = 0$), which is different from the case of single-photon-subtracted STS with a condition $n_c < \sinh^2 \lambda$ [32], but similar to the case of single-photon-added/subtracted squeezed vacuum [28, 37].

Using Equations (32)-(34) we show the plots of WF in the phase space in Fig. 3 for the squeezing parameter ($\lambda = 0.3$) with different photon-added numbers m and average number n_c of the thermal state. One can see clearly that there is some negative region of the WF in the phase space which implies the nonclassicality of this state. In addition, the squeezing effect in one of the quadratures is clear in the plots (see Figure 3(a)), which is another evidence of the nonclassicality of this state. The WF has its minimum value for $m = 1, 3$ at the center of phase space ($\alpha = 0$) (see Fig. 2(a) and (d)). The case is not true for $m = 2$ (see Fig. 2(c)). For $m = 2$, there are two negative regions of the WF, which differs from the case of single PASTS. In addition, the negative region of WF gradually decreases with the increasement of n_c , but not disappear for $m = 1$.

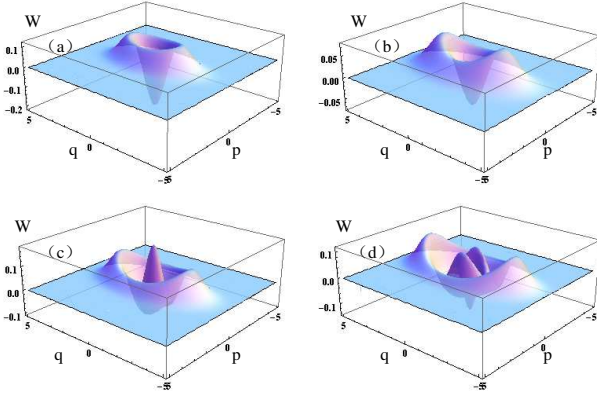


FIG. 3: (Color online) Wigner function distributions $W(\alpha, \alpha^*)$ of PASTS with $\lambda = 0.3$ for different n_c and m values (a) $n_c = 0.1, m = 1$; (b) $n_c = 0.5, m = 1$; (c) $n_c = 0.1, m = 2$; (d) $n_c = 0.1, m = 3$.

V. DECOHERENCE OF PASTS IN THERMAL ENVIRONMENT

In this section, we consider how this single-mode PASTS evolves at the presence of thermal environment. In thermal channel, the evolution of the density matrix for the m -PASV can be described by [38]

$$\begin{aligned} \frac{d\rho}{dt} = & \kappa(\mathcal{N} + 1)(2apa^\dagger - a^\dagger a\rho - \rho a^\dagger a) \\ & + \kappa\mathcal{N}(2a^\dagger \rho a - aa^\dagger \rho - \rho aa^\dagger), \end{aligned} \quad (38)$$

where κ represents the dissipative coefficient and \mathcal{N} denotes the average thermal photon number of the environment. When $\mathcal{N} = 0$, Eq.(38) reduces to the master equation describing the photon-loss channel. The evolution formula of WF of the PASV can be derived as follows [39]

$$W(\eta, \eta^*, t) = \frac{2}{(2\mathcal{N} + 1)\mathcal{T}} \int \frac{d^2\alpha}{\pi} W(\alpha, \alpha^*, 0) e^{-2\frac{|\eta - \alpha e^{-\kappa t}|^2}{(2\mathcal{N} + 1)\mathcal{T}}}, \quad (39)$$

where $W(\alpha, \alpha^*, 0)$ is the WF of the initial state, and $\mathcal{T} = 1 - e^{-2\kappa t}$. Thus, in thermal channel, the WF at any time can be obtained by performing the integration when the initial WF is known.

In a similar way to deriving Eq.(32), substituting Eqs.(32)-(34) into Eq.(39) and using the generating function of single-variable Hermite polynomials (8), we finally obtain (see Appendix B)

$$W(\eta, \eta^*, t) = F_m(\eta, \eta^*, t) W_0(\eta, \eta^*, t), \quad (40)$$

where

$$\begin{aligned} W_0(\eta, \eta^*, t) = & \frac{2/(2n_c + 1)}{\pi(2\mathcal{N} + 1)\mathcal{T}\sqrt{G}} \\ & \times \exp\left[-\Delta_1|\eta|^2 + \frac{g_2 g_3^2}{G}(\eta^{*2} + \eta^2)\right], \end{aligned} \quad (41)$$

$$F_m(\eta, \eta^*, t) = C_{am}^{-1} \sum_{l=0}^m \frac{(m!)^2 \chi^l \Delta_2^{m-l}}{l! [(m-l)!]^2} \left| H_{m-l}\left(\frac{-i\omega/2}{\sqrt{\Delta_2}}\right) \right|^2, \quad (42)$$

and

$$\begin{aligned} g_0 &= \frac{\cosh 2\lambda}{2n_c + 1}, \quad g_1 = \frac{n_c + \cosh^2 \lambda}{2n_c + 1}, \\ g_2 &= \frac{\sinh 2\lambda}{2n_c + 1}, \quad g_3 = \frac{2e^{-\kappa t}}{(2\mathcal{N} + 1)\mathcal{T}}, \end{aligned} \quad (43)$$

as well as

$$\begin{aligned} G &= (2g_0 + g_3 e^{-\kappa t})^2 - 4g_2^2, \\ \Delta_1 &= g_3 e^{\kappa t} - \frac{g_3^2}{G} (2g_0 + g_3 e^{-\kappa t}), \\ \Delta_2 &= \frac{g_2}{G} (g_3 e^{-\kappa t}/2 - 1)^2, \\ \omega &= \frac{2g_3}{g_3 e^{-\kappa t} - 2} (2\Delta_2 \eta^* + \chi \eta), \\ \chi &= \frac{2 - g_3 e^{-\kappa t}}{G} \left(g_0 + g_1 g_3 e^{-\kappa t} + \frac{1}{(2n_c + 1)^2} \right). \end{aligned} \quad (44)$$

Equation (40) is just the analytical expression of WF for PASTS in the thermal channel. It is obvious that the WF loses its Gaussian property due to the presence of single-variable Hermite polynomials. It is interesting to notice that $W_0(\eta, \eta^*, t)$ is actually the WF of squeezed thermal state in thermal channel corresponding to the case without photon addition ($m = 0$), $F_0(\eta, \eta^*, t) = 1$; while $F_m(\eta, \eta^*, t)$ is just the non-Gaussian contribution from photon-addition. The partial negativity of WF is fully determined by that of $F_m(\eta, \eta^*, t)$.

In particular, at the initial time ($t = 0$), noting $(2\mathcal{N} + 1)\mathcal{T}\sqrt{G} \rightarrow 2$, $g_3^2/G \rightarrow 1$, and $\Delta_1 \rightarrow 2g_0$, $\Delta_2 \rightarrow \sinh 2\lambda/[4(2n_c + 1)]$, $\chi \rightarrow -(\cosh^2 \lambda + n_c)/(2n_c + 1)$, as well as $\omega/(2i\sqrt{\Delta_2}) \rightarrow \bar{\gamma} = [\eta^* \sinh 2\lambda - 2\eta(\cosh^2 \lambda + n_c)]/\{i[(2n_c + 1) \sinh 2\lambda]^{1/2}\}$, Eqs.(41) and (42) just do reduce to Eqs.(33) and (34), respectively, i.e., the WF of the PASTS. On the other hand, when $\kappa t \rightarrow \infty$, noticing that $\mathcal{T} \rightarrow 1$, $G \rightarrow 4/(2n_c + 1)^2$, $\Delta_1 \rightarrow 2/(2\mathcal{N} + 1)$, $\omega/(2i\sqrt{\Delta_2}) \rightarrow 0$, $\Delta_2 \rightarrow \frac{1}{4}(2n_c + 1) \sinh 2\lambda$, and $\chi \rightarrow n_c \cosh 2\lambda + \cosh^2 \lambda$, as well as $H_m(0) = (-1)^j \frac{m!}{j!} \delta_{m,2j}$, then Eq.(40) becomes $W(\eta, \eta^*, \infty) = 1/[\pi(2\mathcal{N} + 1)] \exp[-2|\eta|^2/(2\mathcal{N} + 1)]$, a Gaussian distribution, which is independent of photon-addition number m and corresponds to the WF of thermal state with mean thermal photon number \mathcal{N} . This indicates that the system state reduces to a thermal state with mean photon number \mathcal{N} after an enough long time interaction with the environment.

In addition, for the case of $m = 1$, corresponding to the case of SPASTS, Eq. (40) just becomes

$$W_1(\eta, \eta^*, t) = C_{a1}^{-1} W_0(\eta, \eta^*, t) \left(|\omega|^2 + \chi \right). \quad (45)$$

It is obvious that when $F_1(\eta, \eta^*, t) < 0$, the WF of SPASTS in thermal channel has its negative distribution

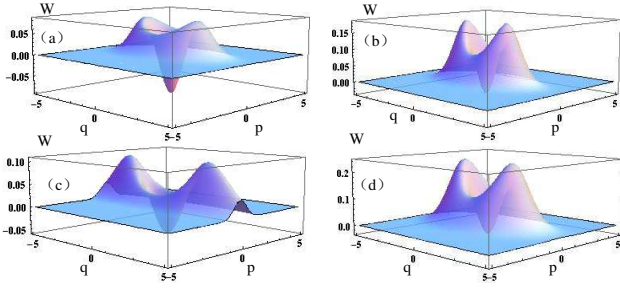


FIG. 4: (Color online) Wigner function distributions $W(\alpha, \alpha^*)$ of PASTS with $m = 1$, $n_c = 0.3$ for different \mathcal{N} , λ and κt values (a) $\mathcal{N} = 0.2$, $\lambda = 0.3$, $\kappa t = 0.05$; (b) $\mathcal{N} = 0.2$, $\lambda = 0.3$, $\kappa t = 0.2$; (c) $\mathcal{N} = 0.2$, $\lambda = 0.8$, $\kappa t = 0.05$; (d) $\mathcal{N} = 2$, $\lambda = 0.3$, $\kappa t = 0.05$.

in phase space. At the center of phase space $\eta = \eta^* = 0$, the WF of SPASTS always has the negative values when $\chi < 0$, i.e., $(2 - g_3 e^{-\kappa t}) / (2g_0 + g_3 e^{-\kappa t} - 2g_2) < 0$ (note $2g_0 + g_3 e^{-\kappa t} - 2g_2 > 0$) leading to the following condition:

$$\kappa t < \kappa t_c = \frac{1}{2} \ln \frac{2\mathcal{N} + 2}{2\mathcal{N} + 1}, \quad (46)$$

which is independent of the squeezing parameter λ and the average photon number n_c of thermal state, there always exist negative region for WF in phase space and the WF of PASTS is always positive in the whole phase space when κt exceeds the threshold value κt_c . Due to this and from Eq. (46), we can see how the thermal noise shortens the threshold value of the decay time. Comparing to the time threshold value of SPSSTS [32] with the identical squeezed thermal state to that of SPASTS,

$$\kappa t_{cs} = \frac{1}{2} \ln \left[1 - \frac{2n_c + 1}{2\mathcal{N} + 1} \frac{n_c - \sinh^2 \lambda}{n_c \cosh 2\lambda + \sinh^2 \lambda} \right], \quad (47)$$

one can find a difference of $e^{2\kappa t_c} - e^{2\kappa t_{cs}}$:

$$e^{2\kappa t_c} - e^{2\kappa t_{cs}} = \frac{2n_c(n_c + 1)}{(2N + 1)(n_c \cosh 2\lambda + \sinh^2 \lambda)}, \quad (48)$$

which implies that the decoherence time of SPASTS is longer than that of SPSSTS. In this sense, the photon-addition Gaussian states present more robust contrast to decoherence than photon-subtraction ones.

In Fig.3, the WFs of PASTS with $m = 1$ and $n_c = 0.3$ are depicted in phase space for several different \mathcal{N} , λ and κt values. It is easy to see that the negative region of WF gradually diminishes as the time κt increases (see Fig.3 (a) and (b)). In addition, the partial negativity of WF decreases gradually as \mathcal{N} (or λ) increases for a given time (see Fig.3 (c) and (d)). The squeezing effect in one of the quadrature is shown in Fig.4(c). For the case of large squeezing value λ and small n_c and \mathcal{N} values, the single-PASTS becomes similar to a Schrodinger cat state. The WF becomes Gaussian with the time evolution.

VI. NON-GAUSSIANITY MEASURE FOR PASTS

As well known, non-Gaussian operators (such as photon-adding/subtracting) can improve the nonclassicality and entanglement between Gaussian states [12, 13]. One reason of such an enhancement is their amount of non-Gaussianity [40, 41]. Recently, an experimentally accessible criterion has been proposed to measure the degree based on the conditional entropy of the state with a Gaussian reference [42]. Therefore, it is of interest to evaluate the degree of the resulting non-Gaussianity and assess this operation as a resource to obtain non-Gaussian states starting from Gaussian ones. Noting that the STS can be considered as a generalized Gaussian state, thus the fidelity between PASTS and STS may be seen as a non-Gaussianity measure. For this purpose, we define the fidelity by [32]

$$\mathcal{F} = \text{tr}(\rho_s \rho) / \text{tr}(\rho_s^2), \quad (49)$$

where ρ_s and ρ are the STS (a generalized Gaussian state) and the PASTS, respectively.

Noticing $\text{tr}(\rho_s^2) = 1/(2\bar{n}_c + 1)$, and using the formula (C1), we finally obtain (see Appendix C)

$$\mathcal{F} = \frac{m!}{C_{a,m}} K_2^{m/2} P_m \left(\frac{K_1}{\sqrt{K_2}} \right) = \left(\frac{K_2}{A} \right)^{m/2} \frac{P_m(K_1/\sqrt{K_2})}{P_m(\bar{B}/\sqrt{A})}, \quad (50)$$

where

$$K_1 = \frac{n_c(n_c + 1)}{2n_c + 1} \cosh 2\lambda, K_2 = \frac{n_c^2(n_c + 1)^2}{(2n_c + 1)^2} - \frac{\sinh^2 2\lambda}{4}. \quad (51)$$

Eq.(50) is just the analytical expression for the fidelity between PASTS and STS. It is obvious that when $m = 0$ (without photon-addition), $\mathcal{F} = 1$. Comparing to the fidelity \mathcal{F}_s between PSSTS and STS (59) in Ref.[32], one can clearly see that

$$\frac{\mathcal{F}}{\mathcal{F}_s} = \left(\frac{Z}{A} \right)^{m/2} \frac{P_m(H/\sqrt{Z})}{P_m(\bar{B}/\sqrt{A})} = \frac{C_{s,m}}{C_{a,m}}, \quad (52)$$

where $Z = n_c^2 - (2n_c + 1) \sinh^2 \lambda$, $H = n_c \cosh 2\lambda + \sinh^2 \lambda$. Eq.(52) implies that the ratio to fidelities is just that to the normalization factors. In particular, for $m = 1$ (the case of SPASTS), Eq.(50) reduces to

$$\frac{\mathcal{F}}{\mathcal{F}_s} = \frac{n_c \cosh 2\lambda + \sinh^2 \lambda}{n_c \cosh 2\lambda + \cosh^2 \lambda} < 1, \quad (53)$$

from which one can see that $\mathcal{F} < \mathcal{F}_s$, i.e., the amount of non-Gaussianity for SPASTS is larger than that for SPSSTS.

This point is made clear in Fig.5, in which the fidelity \mathcal{F} between PASTS and STS as the function of squeezing parameter λ for different photon-addition number

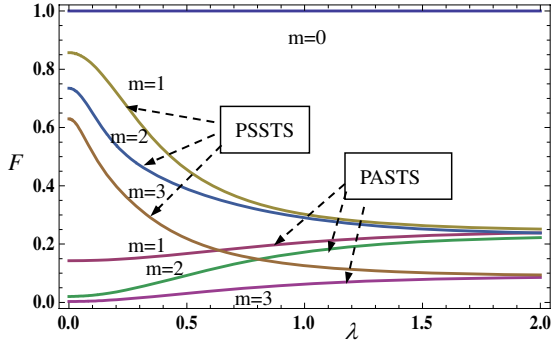


FIG. 5: (Color online) The fidelity \mathcal{F} between PASTS (PSSTS) and STS as the function of squeezing parameter λ for different photon-addition number $m = 0, 1, 2, 3 (n_c = 0.2)$.

m . As a comparison, the fidelity \mathcal{F}_s between PSSTS and STS is also shown in Fig.5, from which one can see that the fidelity decreases as the increment of photon-addition/subtraction number m , as expected. The fidelity \mathcal{F} increases monotonously with the augment of the squeezing parameter λ . However, the case is not true for the fidelity \mathcal{F}_s . For a given m value, the fidelity \mathcal{F} is always smaller than the fidelity \mathcal{F}_s within the region shown in Fig.5. In this sense, the amount of non-Gaussianity for PASTS is larger than that for PSSTS.

VII. CONCLUSIONS

In this paper, we investigate the nonclassical properties and decoherence of single-mode PASTS when evolving under a thermal environment. Based on the fact that squeezed number can be considered as an Hermite polynomial excitation squeezed vacuum, the normally ordering form of PASTS is directly obtained, from which one can expediently calculate some quasi-distributions, such as Q-, P- and Wigner function; And the normalization factor of PASTS is analytically derived, which is just proved to be an m -order Legendre polynomial of the squeezing parameter r and average photon number n_c of the thermal state, a remarkable result. Furthermore, for any photon-added number m -PASTS, the explicit expression of WF is derived, which considered as a product of the WF of STS in thermal channel and a non-Gaussian distribution resulting from photon-addition. It is shown that the WF of SPASTS always has the negative values at the phase space center, which is different from the case of SPSSTS with a condition $n_c < \sinh^2 \lambda$. Then the effects of decoherence to the nonclassicality of PASTS in the thermal channel is also demonstrated according to the compact expression for the WF. The threshold value of the decay time corresponding to the transition of the WF from partial negative to completely positive definite is obtained for SPASTS at the center of phase space. It is found that the WF has always negative value for all parameters r, n_c if the decay time $\kappa t < \kappa t_c = \frac{1}{2} \ln \frac{2N+2}{2N+1}$,

a larger value than that of SPSSTS.

A comparison between the nonclassicality and decoherence of PASTS and PSSTS shows that the photon-addition non-Gaussian states present more robust contrast to decoherence than photon-subtraction ones, which may be due to the amount of non-Gaussianity for SPASTS is larger than that for SPSSTS. On the other hand, in the limit of vanishing squeezing and $n_c = 0$, the PASTS reduces to a single-mode Fock state, remaining non-Gaussian, while the PSSTS becomes Gaussian, as it reduces to the single mode vacuum. Entanglement evaluation investigation for photon-subtracted/added two-mode squeezed thermal state is a future problem.

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Appendix A: Derivation of WF (32) for PASTS

Substituting Eq.(16) into Eq.(31) and using the integration formula (18), we have

$$\begin{aligned} W(\alpha, \alpha^*) &= \frac{(-1)^m C_{am}^{-1} e^{2|\alpha|^2}}{\tau_1 \tau_2} \int \frac{d^2 \beta}{\pi^2} |\beta|^{2m} \exp \left[-(1+B)|\beta|^2 \right. \\ &\quad \left. + 2(\alpha\beta^* - \alpha^*\beta) + \frac{C}{2}(\beta^{*2} + \beta^2) \right] \\ &= \frac{C_{am}^{-1} e^{2|\alpha|^2}}{\tau_1 \tau_2} \frac{\partial^{2m}}{\partial s^m \partial t^m} \int \frac{d^2 \beta}{\pi^2} \exp \left[-(1+B)|\beta|^2 \right. \\ &\quad \left. + (2\alpha + s)\beta^* - (2\alpha^* + t)\beta + \frac{C}{2}(\beta^{*2} + \beta^2) \right]_{s=t=0} \\ &= W_0(\alpha, \alpha^*) F_m(\alpha, \alpha^*), \end{aligned} \quad (\text{A1})$$

where we have set

$$W_0(\alpha, \alpha^*) = \frac{\sqrt{A_1}}{\pi \tau_1 \tau_2} \exp \left[A_2(\alpha^2 + \alpha^{*2}) - 2A_3|\alpha|^2 \right], \quad (\text{A2})$$

$$\begin{aligned} F_m(\alpha, \alpha^*) &= C_{am}^{-1} \frac{\partial^{2m}}{\partial s^m \partial t^m} \exp \left[\frac{A_2}{4}(s^2 + t^2) - \frac{A_4}{2}st \right. \\ &\quad \left. + (A_2\alpha^* - A_4\alpha)t + (A_2\alpha - A_4\alpha^*)s \right]_{s=t=0}, \end{aligned} \quad (\text{A3})$$

and

$$\begin{aligned} A_1 &= \frac{1}{(1+B)^2 - C^2} = \frac{A}{(2n_c + 1)^2}, \\ A_2 &= \frac{2C}{(1+B)^2 - C^2} = \frac{\sinh 2\lambda}{2n_c + 1}, \\ A_3 &= \frac{2(B+1)}{(1+B)^2 - C^2} - 1 = \frac{\cosh 2\lambda}{2n_c + 1}, \\ A_4 &= \frac{2(B+1)}{(1+B)^2 - C^2} = A_3 + 1 = 2 \frac{n_c + \cosh^2 \lambda}{2n_c + 1}. \end{aligned} \quad (\text{A4})$$

Substituting Eq.(A3) into Eq.(A2) yields Eq.(33), i.e., the WF of squeezed thermal state.

Further expanding the exponential term st included in (A3) into sum series, and using the generating function of single-variable Hermite polynomials [27],

$$H_n(x) = \frac{\partial^n}{\partial t^n} \exp(2xt - t^2) \Big|_{t=0}, \quad (\text{A5})$$

which leads to

$$\begin{aligned} & \frac{\partial^n}{\partial t^n} \exp(At + Bt^2) \Big|_{t=0} \\ &= (i\sqrt{B})^n H_n[A/(2i\sqrt{B})] \\ &= (-i\sqrt{B})^n H_n[A/(-2i\sqrt{B})], \end{aligned} \quad (\text{A6})$$

thus we can see

$$\begin{aligned} F_m(\alpha, \alpha^*) &= C_{am}^{-1} \sum_{l=0}^{\infty} \frac{(-A_4)^l}{2^l l!} \frac{\partial^{2m}}{\partial s^m \partial \tau^m} s^l t^l \\ &\times \exp \left[\frac{A_2}{4} (s^2 + t^2) + \gamma t + \gamma^* s \right]_{s=t=0} \\ &= C_{am}^{-1} \sum_{l=0}^{\infty} \frac{(-A_4)^l}{2^l l!} \frac{\partial^{2l}}{\partial \gamma^l \partial \gamma^{*l}} \frac{\partial^{2m}}{\partial s^m \partial \tau^m} \\ &\times \exp \left[\frac{A_2}{4} (s^2 + t^2) + \gamma t + \gamma^* s \right]_{s=t=0} \\ &= \frac{A_2^m}{2^{2m}} C_{am}^{-1} \sum_{l=0}^{\infty} \frac{(-A_4)^l}{2^l l!} \frac{\partial^{2l}}{\partial \gamma^l \partial \gamma^{*l}} |H_m(\bar{\gamma})|^2, \end{aligned} \quad (\text{A7})$$

where $\gamma = A_2\alpha^* - A_4\alpha$, and $\bar{\gamma} = \gamma/(i\sqrt{A_2})$, i.e.,

$$\bar{\gamma} = \frac{\alpha^* \sinh 2\lambda - 2\alpha (\cosh^2 \lambda + n_c)}{i\sqrt{(2n_c + 1) \sinh 2\lambda}}, \quad (\text{A8})$$

Then using the recurrence relation of $H_n(x)$,

$$\frac{d}{dx} H_n(x) = \frac{2^n n!}{(n-1)!} H_{n-1}(x), \quad (\text{A9})$$

Eq.(A7) becomes

$$\begin{aligned} F_m(\alpha, \alpha^*) &= \frac{A_2^m}{2^{2m}} C_{am}^{-1} \sum_{l=0}^{\infty} \frac{(-A_4/A_2)^l}{2^l l!} \\ &\times \frac{\partial^{2l}}{\partial \bar{\gamma}^l} H_m(\bar{\gamma}) \frac{\partial^{2l}}{\partial \bar{\gamma}^{*l}} H_m(\bar{\gamma}^*) \\ &= \frac{A_2^m}{2^{2m}} C_{am}^{-1} \sum_{l=0}^m \frac{(m!)^2 (-2A_4/A_2)^l}{l! [(m-l)!]^2} |H_{m-l}(\bar{\gamma})|^2. \end{aligned} \quad (\text{A10})$$

Substituting Eq.(A4) into Eq.(A10) yields Eq.(34). Thus we complete the derivation of WF Eq.(32) by combing Eqs. (A2) and (A10).

Appendix B: Derivation of WF (40) for PASTS in thermal channel

Substituting Eqs.(32)-(34) into Eq.(39), we have

$$\begin{aligned} W(\eta, \eta^*, t) &= \frac{C_{am}^{-1} g_3 e^{\kappa t}}{\pi (2n_c + 1)} e^{-g_3 e^{\kappa t} |\eta|^2} \frac{\partial^{2m}}{\partial s^m \partial \tau^m} \\ &\times \exp \left[\frac{g_2}{4} (s^2 + \tau^2) - g_1 s \tau \right] \\ &\times \int \frac{d^2 \alpha}{\pi} \exp \left[- (2g_0 + g_3 e^{-\kappa t}) |\alpha|^2 \right. \\ &\quad \left. + (g_3 \eta^* + g_2 s - 2g_1 \tau) \alpha \right. \\ &\quad \left. + (g_3 \eta + g_2 \tau - 2g_1 s) \alpha^* + g_2 (\alpha^2 + \alpha^{*2}) \right]_{s=\tau=0}, \end{aligned} \quad (\text{B1})$$

where we have set

$$\begin{aligned} g_0 = A_3 &= \frac{\cosh 2\lambda}{2n_c + 1}, \quad g_1 = \frac{A_4}{2} = \frac{n_c + \cosh^2 \lambda}{2n_c + 1}, \\ g_2 = A_2 &= \frac{\sinh 2\lambda}{2n_c + 1}, \quad g_3 = \frac{2e^{-\kappa t}}{(2\mathcal{N} + 1)\mathcal{T}}. \end{aligned} \quad (\text{B2})$$

Further using the integration (18), Eq.(B1) can be put into the form

$$W(\eta, \eta^*, t) = F_m(\eta, \eta^*, t) W_0(\eta, \eta^*, t), \quad (\text{B3})$$

where $W_0(\eta, \eta^*, t)$ is defined in Eq.(41), and

$$\begin{aligned} F_m(\eta, \eta^*, t) &= C_{am}^{-1} \frac{\partial^{2m}}{\partial s^m \partial \tau^m} \exp [\Delta_2 (s^2 + \tau^2) \\ &\quad + \omega \tau + \omega^* s + \chi s \tau]_{s=\tau=0}, \end{aligned} \quad (\text{B4})$$

here (Δ_2, ω, χ) are defined in Eq. (44). In a similar way to deriving Eq. (32), we can further insert Eq. (B4) into Eq. (42).

Appendix C: Derivation of fidelity (50) between PASTS and STS

The fidelity ($\text{tr}(\rho_s \rho)$) can be calculated as the overlap between the two WFs:

$$\text{tr}(\rho_s \rho) = 4\pi \int d^2 \alpha W_0(\alpha, \alpha^*) W_\rho(\alpha, \alpha^*), \quad (\text{C1})$$

where $W_0(\alpha, \alpha^*)$ is the WF of squeezed thermal state ρ_s . Using Eq.(32) we may express Eq.(C1) as

$$\text{tr}(\rho_s \rho) = 4\pi \int F_m(\alpha, \alpha^*) W_0^2(\alpha, \alpha^*) d^2 \alpha. \quad (\text{C2})$$

Then employing Eqs.(32) and (A2),(A3) as well as the integration formula (18), we can put Eq.(C2) into the following form:

$$\begin{aligned} \text{tr}(\rho_s \rho) &= \frac{4C_{am}^{-1}}{(2n_c + 1)^2} \frac{\partial^{2m}}{\partial s^m \partial \tau^m} \exp \left[\frac{g_2}{4} (s^2 + \tau^2) - g_1 s \tau \right] \\ &\int \frac{d^2 \alpha}{\pi} \exp \left[-4g_0 |\alpha|^2 + 2g_2 (\alpha^2 + \alpha^{*2}) \right] \\ &\quad + (g_2 s - 2g_1 \tau) \alpha + (g_2 \tau - 2g_1 s) \alpha^*]_{s=\tau=0} \\ &= \frac{C_{am}^{-1}}{2n_c + 1} \frac{\partial^{2m}}{\partial s^m \partial \tau^m} \exp [K_1 s \tau + K_0 (s^2 + \tau^2)]_{s=\tau=0}, \end{aligned} \quad (\text{C3})$$

where K_1 is defined in Eq.(51), and

$$K_0 = \frac{2n_c^2 + 2n_c + 1}{4(2n_c + 1)} \sinh 2\lambda. \quad (\text{C4})$$

Similarly to deriving Eq.(20), we have

$$\begin{aligned} & \frac{\partial^{2m}}{\partial s^m \partial \tau^m} \exp [K_0 (k^2 + t^2) + K_1 kt] \Big|_{k=t=0} \\ &= m! K_2^{m/2} P_m \left(K_1 / \sqrt{K_2} \right), \end{aligned} \quad (\text{C5})$$

and $K_2 \equiv K_1^2 - 4K_0^2$ given in Eq.(51), which leads to Eq.(50).

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